On the design of wholesale electricity markets under uncertainty

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***Abstract*—We propose a market mechanism for an electricity market under uncertainty, comprising of dispatchable generators, renewable power producers, and load-serving entities. The single-settlement market mechanism consists of a stochastic economic dispatch and a contingent nodal pricing scheme. We show that the market mechanism is efficient, revenue adequate in all scenarios of available renewable supply, and supports a competitive equilibrium. The proposed design is illustrated through the analysis of a copperplate power system. Finally, it is compared against existing market designs in the literature.**

***Index Terms*—Electricity markets, Stochastic economic dispatch, Renewable integration**

I. Introduction

Renewable energy resources like wind and solar are uncertain (difficult to forecast even over short horizons), intermittent (exhibit large fluctuations in power output over short timescales), and largely uncontrollable (cannot be dispatched on demand). Such variability makes it challenging to balance demand and supply across a transmission-constrained power network at all times. Integrating renewable resources into the power system thus requires the design of (1) reliable strategies to procure power under uncertainty at minimum cost, and (2) efficient and equitable market mechanisms to compensate the load serving entities and power producers in a deregulated market environment. The focus of the current paper is on the latter. Of interest are payment schemes that reflect both the production cost as well as the cost of balancing uncertain supply.

Balancing demand and supply typically requires forward planning. Such needs arise due to the finite ramping capabilities of some generators. For example, nuclear power plants and certain coal-fired thermal generators cannot quickly alter their power output to meet fluctuations in demand and/or renewable supply in real time. Though fast ramping generators, e.g., natural gas turbines, can absorb some of the variability, these resources are usually more expensive. In current electricity markets, the planning or the forward stage typically corresponds to a day before the time of power delivery. At the forward stage, the system operator (SO) is unaware of the available renewable supply and/or demand, but has access to a forecast of said variables. The SO utilizes the available forecasts to: (1) solve the unit commitment problem, where it designates which power plants will be available to supply power in real time, and (2) clear the market to compute a forward dispatch, that defines (a) the set points for generators with limited ramping capabilities, and (b) forward payments to each market participant. Another market is cleared, between five minutes to an hour prior to the time of physical power delivery. It is reasonable to assume that all uncertainty has realized, when this second market is cleared. This so-called real time market balances any attending deviations of uncertain variables from their day-ahead forecasts, and compensates market participants for said deviations. Such a two-settlement design is common in practice. For example, see [1].

The dispatch in the forward stage typically does not consider the cost of balancing in real time and, hence, is myopic in nature. Imbalances arising in real time are addressed by using fast ramping reserves. When forecast errors are small, the resulting cost of such balancing is relatively low. Errors in day-ahead forecasts remain as low as 1-3% when there is little or no renewable supply in the grid and the principal source of uncertainty is demand. As available renewable supply increases, however, said errors in day-ahead forecasts can be as large as 15-20% (mean absolute error) [2]. As a result, high levels of renewable penetration will likely result in significant increase in reserve requirements and system operational costs [3], [4]. Ideally, one should optimize in the forward stage against the expected cost of balancing in real-time. Then, the optimization problem in the forward stage can be formulated as a stochastic program. See [5]–[12] for examples.

Two kinds of payment schemes have been proposed in the literature. One pays for both power consumption/production, as well as reserves procured forward, anticipating the balancing needs in real time [11], [12]. Another only defines payments for power, that is either procured forward and/or actually consumed/produced in real time [13], [14]. Of particular interest is the formulation in [13], where the authors design payments for power to accompany a stochastic programming based dispatch. While their approach is guaranteed to be revenue adequate in expectation, it is shown to be revenue inadequate in certain scenarios of available renewable supply.

The current paper offers a market mechanism that prices power within a stochastic programming framework. We show that the proposed mechanism (1) results in an efficient dispatch, (2) is revenue adequate in all scenarios of available renewable supply, and (3) supports a competitive equilibrium. In a sense, the proposed design addresses the drawbacks of existing designs. However, it results in increased complexity of market operations, specifically in terms of the information, the SO needs to communicate to each market participant.

The paper is organized as follows. We begin in Section II by defining notation important in the sequel. Then, Section III describes the model of the power system and the market participants. In Section IV, we discuss two existing electricity market designs, and in Section V, present the proposed design. In Section VI, we illustrate the proposed mechanism through an example, and in Section VII, compare it against the two

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existing designs. Finally, Section VIII concludes the paper. The proofs of the results are omitted for space constraints.

II. Notation

Let $\mathbf{R}$ denote the set of real numbers. Let $\mathbf{R}_+$ (resp. $\mathbf{R}_{++}$) denote the set of nonnegative (resp. positive) real numbers. For a vector or matrix X, let $X^\top$ denote its transpose. For a random variable X, let $\mathbb{E}[X]$ denote the expectation of X. For an event X, let $\mathbf{P}\{X\}$ denote the probability of event X. For any set $\mathcal{K}$, let $|\mathcal{K}|$ denotes its cardinality. Let $\mathbf{1}$ denote the vector of all ones of appropriate size. For $x \in \mathbf{R}$, let $x^+ := \max\{x, 0\}$. For a decision variable x in an optimization problem, we shall refer to the variable at optimality as x^*.

In the sequel, we shall study a collection of maps of the form $x_k : \Omega \to \mathbf{R}, k \in \mathcal{K}$, where Ω is an arbitrary set, and $\mathcal{K}$ is an arbitrary index set. For such a collection, we use the following notation.

$$\begin{aligned} x(\omega) &:= (x_k(\omega), k \in \mathcal{K}), \\ x &:= (x_k(\omega), k \in \mathcal{K}, \omega \in \Omega), \end{aligned} \tag{1}$$

where $(,)$ is identified with a vector, in which comma-separated elements are concatenated together.

III. Preliminaries

We begin by describing the power system model and the wholesale electricity market. The market consists of a collection of consumers, producers, and a system operator (SO). The consumers are load-serving entities that represent the retail customers they serve. Consumers' demands are modeled as flexible and price-responsive. The mathematical model also accommodates inelastic fixed demand as a special case. We consider two types of producers: (i) one who owns and operates a dispatchable generator, e.g., a fossil fuel based one, and (ii) the other that owns and operates a variable renewable resource, e.g., a wind or a solar farm. The SO implements a centralized market mechanism that defines: (1) the dispatch, or the amount of power each consumer/producer demands/supplies, and (2) the payments for each consumer/producer. The focus of the current paper is on the design of such a market mechanism.

A. Power system model

Consider an electric power system described by an undirected graph on a collection of nodes, denoted by the set $\mathcal{N}$. The vector of (directional) power flows along the m transmission lines is related to the vector of nodal power injections $x \in \mathbf{R}^{|\mathcal{N}|}$ by a linear map Hx, where $H \in \mathbf{R}^{2m \times |\mathcal{N}|}$ is commonly referred to as the shift factor matrix [15]. The above relation is derived using a linear approximation (commonly known as the DC approximation) of the Kirchhoff's laws. Let $f \in \mathbf{R}_+^{2m}$ denote the capacities of the transmission lines. Then, the transmission-constrained power network defines the following region of feasible nodal power injections

$$\mathcal{P} := \left\{ x \in \mathbf{R}^{|\mathcal{N}|} \mid Hx \le f,\ \mathbf{1}^\top x = 0 \right\}, \tag{2}$$

where $\mathbf{1}^\top x = 0$ represents the balance of demand and supply across the network. The set $\mathcal{P} \subset \mathbf{R}^N$ is often referred to as the *injection polytope*.

B. Modeling uncertainty

The electricity market is modeled via a two-period ($t = 0, 1$) economy. We model the uncertainty according to an underlying probability space $(\Omega, \mathcal{F}, \mathbf{P})$. Here, the set Ω denotes the set of all possible scenarios at $t = 1$, $\mathcal{F}$ denotes a suitably defined σ-algebra on Ω, and $\mathbf{P}$ is a probability measure. In the sequel, we encode the uncertainty in available renewable supply[1] across the network at $t = 1$ through suitably defined functions (random variables) on Ω.

Assumption 1 (Uncertainty set)**.** *The set Ω is finite, and $\mathbf{P}\{\omega\} > 0$ for each $\omega \in \Omega$. At $t = 0$, each consumer, producer, and the system operator holds the belief that the distribution on Ω is described by $\mathbf{P}$.*

We shall refer to $t = 0$ as the 'day-ahead stage', and $t = 1$ as 'real-time'.

C. Market participants

Consider a collection $\mathcal{I}$ of consumers, and a collection $\mathcal{J}$ of producers. Each consumer (resp. producer) demands (resp. supplies) power at a particular node $n \in \mathcal{N}$. Denote the set of consumers (resp. producers) demanding (resp. supplying) power at node n by $\mathcal{I}(n)$ (resp. $\mathcal{J}(n)$).

The consumers: Consumer $i \in \mathcal{I}$ demands $d_i(\omega)$ amount of power in scenario $\omega \in \Omega$ at $t = 1$. For simplicity, the range of possible demands is assumed to be independent of ω, succinctly represented as $d_i(\omega) \in \left[\underline{d}_i, \overline{d}_i\right]$. Consumer i derives a utility of $u_i(d_i(\omega))$, when consuming $d_i(\omega)$. An inelastic demand can be modeled by letting $\underline{d}_i = \overline{d}_i$. The utility function $u_i : \left[\underline{d}_i, \overline{d}_i\right] \to \mathbf{R}_+$ is assumed to be smooth, concave, nondecreasing, and independent of ω.

The producers: Producer $j \in \mathcal{J}$ supplies $g_j(\omega)$ amount of power in scenario $\omega \in \Omega$. The available capacity of production in scenario ω is defined by $\left[\underline{g}_j(\omega), \overline{g}_j(\omega)\right]$, that satisfies[2]

$$0 \le \underline{g}_j(\omega) \le \overline{g}_j(\omega) \le \overline{g}_j^{\text{cap}},$$

where $\overline{g}_j^{\text{cap}}$ denotes the nameplate capacity of production for producer j. For a dispatchable generator, $\underline{g}_j(\omega)$ and $\overline{g}_j(\omega)$ are the same for all scenarios $\omega \in \Omega$. For a renewable power producer, the available range of production varies with ω. Each generator incurs a cost of production, $c_j(g_j(\omega))$. Assume $c_j : [0, \overline{g}_j^{\text{cap}}] \to \mathbf{R}_+$ to be smooth, convex, nondecreasing, and independent of ω. Most dispatchable generators also have a finite ramping capability. This is modeled by the constraint $\left|g_j^0 - g_j(\omega)\right| \le \ell_j$, where g_j^0 defines a generator set point, and ℓ_j defines its ramping limit.

D. Social welfare maximization

Call the collection of consumptions/productions of all consumers/producers as a *dispatch*. Define the *social welfare* of

[1] One can also model contingencies arising due to the failures of dispatchable generators in Ω.

[2] Essentially, $\underline{g}_j : \Omega \to \mathbf{R}$, and $\overline{g}_j : \Omega \to \mathbf{R}$ are random variables for each $j \in \mathcal{J}$.

a dispatch, given by a vector of consumptions $x \in \mathbf{R}^{|\mathcal{I}|}$ and a vector of productions $y \in \mathbf{R}^{|\mathcal{J}|}$, as

$$\mathsf{SW}(x, y) := \sum_{i\in\mathcal{I}} u_i(x_i) - \sum_{j\in\mathcal{J}} c_j(y_j). \tag{3}$$

Recall that in scenario $\omega \in \Omega$, the consumption of consumer $i \in \mathcal{I}$ is $d_i(\omega)$, and the production of producer $j \in \mathcal{J}$ is $g_j(\omega)$. Using the notation in (1), a *dispatch plan* at $t = 0$ is then given by d, g. The SO would ideally like to compute a dispatch plan that maximizes the expected social welfare, operating within the constraints of the individual market participants, and the underlying power grid. In particular, the SO would like to solve the following stochastic program.

Stochastic economic dispatch (SED):

$$\begin{aligned}
\text{maximize}\quad & \mathbb{E}\left[\mathsf{SW}\left(d(\omega), g(\omega)\right)\right], \\
\text{subject to}\quad & \underline{d}_i \le d_i(\omega) \le \overline{d}_i, && \text{(4a)}\\
& \underline{g}_j(\omega) \le g_j(\omega) \le \overline{g}_j(\omega), && \text{(4b)}\\
& g_j^0 - \ell_j \le g_j(\omega) \le g_j^0 + \ell_j, && \text{(4c)}\\
& q_n(\omega) = \sum_{j\in\mathcal{J}(n)} g_j(\omega) - \sum_{i\in\mathcal{I}(n)} d_i(\omega), && \text{(4d)}\\
& q(\omega) \in \mathcal{P}, && \text{(4e)}\\
& \text{for each } i \in \mathcal{I}, j \in \mathcal{J}, n \in \mathcal{N}, \omega \in \Omega,
\end{aligned}$$

over the variables $d \in \mathbf{R}^{|\mathcal{I}||\Omega|}$, $g \in \mathbf{R}^{|\mathcal{J}||\Omega|}$, and $q \in \mathbf{R}^{|\mathcal{N}||\Omega|}$. The variable $q_n(\omega)$ is the power injection at node $n \in \mathcal{N}$ in scenario $\omega \in \Omega$. Using the notation in (1), $q(\omega)$ defines the vector of nodal power injections in scenario ω. Further, g^0 denotes the vector of generator set points.

Costs c are convex, utilities u are concave, and the constraints (4a) – (4e) are linear. Hence, SED in (4) is a convex program [16]. The dimension of the problem grows with $|\Omega|$. When the number of scenarios is large, solving (4) becomes computationally challenging. Addressing the computational burdens in solving such a stochastic program is an interesting question, in of itself. However, the focus of the current paper is on market design. We shall assume that SED can be solved optimally.[3]

Assumption 2. *The stochastic economic disptch (SED) problem is feasible.*

The above assumption implies that enough units are committed prior to solving the SED problem, such that demand and supply can be balanced in all scenarios $\omega \in \Omega$, within the constraints of the market participants and the operational limits of the underlying grid.

Definition 1 (Efficient dispatch plan)**.** *Consumption plans d and production plans g constitute an efficient dispatch plan, if there exists g^0 and q, such that d, g, g^0, and q solve the stochastic economic dispatch problem* (4).

[3]When Ω is compact, and not finite, the optimal solutions are maps on subsets of the Euclidean space. Then, SED is an infinite-dimensional problem. Optimal maps or plans for such problems, in general, cannot be computed efficiently. One can optimize over a linear space of maps on Ω, spanned by a suitable set of basis functions, to obtain approximately optimal maps.

IV. Existing Market Designs

An electricity market mechanism comprises of: (1) a dispatch scheme that defines the amount of power each consumer/producer consumes/produces, and (2) a payment scheme for compensating the market participants. We now describe two electricity market designs under uncertainty, within the context of our model. The first design uses a certainty equivalent dispatch scheme, and is widely used in practice. The second design – proposed in [13] – associates prices with a stochastic dispatch, that is similar to SED in (4). For each design, we delineate the dispatch scheme, and the associated payments. We also describe their known limitations, which motivates our design in the next section.

A. Pricing a certainty equivalent dispatch

The market mechanism based on certainty equivalent dispatch consists of two steps: one at $t = 0$, and another at $t = 1$. At $t = 0$, the SO replaces all random variables by their certainty equivalents, and solves

$$\begin{aligned}
\text{maximize}\quad & \mathsf{SW}(D, G), \\
\text{subject to}\quad & \underline{d}_i \le D_i \le \overline{d}_i, && \text{(5a)}\\
& \underline{G}_j \le G_j \le \overline{G}_j, && \text{(5b)}\\
& Q_n = \sum_{j\in\mathcal{J}(n)} G_j - \sum_{i\in\mathcal{I}(n)} D_i, && \text{(5c)}\\
& Q \in \mathcal{P}, && \text{(5d)}\\
& \text{for each } i \in \mathcal{I}, j \in \mathcal{J}, n \in \mathcal{N},
\end{aligned}$$

over the variables $D \in \mathbf{R}^{|\mathcal{I}|}$, $G \in \mathbf{R}^{|\mathcal{J}|}$, and $Q \in \mathbf{R}^{|\mathcal{N}|}$. The optimal solution defines the forward dispatch D^*, G^*. For each node $n \in \mathcal{N}$, let Λ_n^* be the optimal Lagrange multiplier for the power balance constraint in (5c). Then, Λ^* defines the forward nodal prices of power, which in turn determines the following forward payments:

$$\begin{aligned}
&\text{Consumer } i \in \mathcal{I}(n) \text{ pays } \Lambda_n^* D_i^*,\\
&\text{Producer } j \in \mathcal{J}(n) \text{ is paid } \Lambda_n^* G_j^*.
\end{aligned}$$

At $t = 1$, $\omega \in \Omega$ is realized, and the SO solves:

$$\begin{aligned}
\text{maximize}\quad & \mathsf{SW}\left(d(\omega), g(\omega)\right), \\
\text{subject to}\quad & \underline{d}_i \le d_i(\omega) \le \overline{d}_i, && \text{(6a)}\\
& \underline{g}_j(\omega) \le g_j(\omega) \le \overline{g}_j(\omega), && \text{(6b)}\\
& G_j^* - \ell_j \le g_j(\omega) \le G_j^* + \ell_j, && \text{(6c)}\\
& q_n(\omega) = \sum_{j\in\mathcal{J}(n)} g_j(\omega) - \sum_{i\in\mathcal{I}(n)} d_i(\omega), && \text{(6d)}\\
& q(\omega) \in \mathcal{P}, && \text{(6e)}\\
& \text{for each } i \in \mathcal{I}, j \in \mathcal{J}, n \in \mathcal{N},
\end{aligned}$$

over the variables $d(\omega) \in \mathbf{R}^{|\mathcal{I}|}$, $g(\omega) \in \mathbf{R}^{|\mathcal{J}|}$, $q(\omega) \in \mathbf{R}^{|\mathcal{N}|}$. In scenario $\omega \in \Omega$, demand of consumer $i \in \mathcal{I}(n)$ is given by $d_i^*(\omega)$, and the supply of producer $j \in \mathcal{J}(n)$ is given by $g_j^*(\omega)$. For each $n \in \mathcal{N}$, let the optimal Lagrange multiplier for the power balance constraint in (6d) be $\lambda_n^*(\omega)$ in scenario

ω. Then, $\lambda^*(\omega)$ defines the nodal prices of power in real time, which in turn determine the following real time payments:

$$\text{Consumer } i \in \mathcal{I}(n) \text{ pays } \lambda_n^*(\omega)(d_i^*(\omega) - D_i^*),$$
$$\text{Producer } j \in \mathcal{J}(n) \text{ is paid } \lambda_n^*(\omega)(g_j^*(\omega) - G_j^*).$$

The total payment to each market participant is given by the sum of his forward payment and his real time payment. In solving (5), a common practice is to use the certainty equivalents $\underline{G}_j = \mathbb{E}[\underline{g}_j(\omega)]$ and $\overline{G}_j = \mathbb{E}[\overline{g}_j(\omega)]$; for example, see [1], [12]. We refer the reader to [14] for a discussion on alternate definitions for $\underline{G}_j, \overline{G}_j$.

Next, we discuss the limitations of this design. First, consider d^*, g^*, where $d^*(\omega), g^*(\omega)$ solve (6) for each $\omega \in \Omega$. Then, d^*, g^* may not be an efficient dispatch plan. Having ignored the real-time considerations in solving (5), the resulting generator set points G^* – specifically for the dispatchable generators with limited ramping capabilities – can be very different from the generator set points obtained from an optimal solution of (4). As a result, the expected social welfare of this scheme may be lower, when compared to the expected social welfare at the optimal solution of (4). Second, there exists systems for which (4) is feasible, but (6) is infeasible for some scenario $\omega \in \Omega$. As a result, one might need additional generators to be committed during the unit commitment phase in order to ensure feasibility in all scenarios in a certainty equivalent dispatch scheme, when there exists a dispatch plan with less number of committed generators. Third, the dispatch at the forward stage does not take into account the balancing needs in real-time. As a result, the prices, and hence, the payments in the forward stage do not reflect the effect of uncertainty.

B. Pricing a stochastic economic dispatch

Pritchard et al. in [13] suggest solving the following stochastic program at $t = 0$:[4]

$$\begin{aligned}
\text{maximize} \quad & \mathbb{E}\left[\mathsf{SW}\left(d(\omega), g(\omega)\right)\right], \\
\text{subject to} \quad & (4a) - (4e), \\
& q_n^0 = \sum_{j \in \mathcal{J}(n)} g_j^0 - \sum_{i \in \mathcal{I}(n)} d_i^0, \qquad (7a)\\
& q^0 \in \mathcal{P}, \qquad (7b)\\
& \text{for each } i \in \mathcal{I}, j \in \mathcal{J}, \omega \in \Omega,
\end{aligned}$$

where $d^0 \in \mathbf{R}^{|\mathcal{I}|}$, $q^0 \in \mathbf{R}^{|\mathcal{N}|}$. For each $n \in \mathcal{N}$, let the optimal Lagrange multiplier for (7a) be Λ_n^*. Also, for each $\omega \in \Omega$, let the optimal Lagrange multiplier of (4d) be given by $\lambda_n^*(\omega)$. Then, the payments of the market participants are defined as:

$$\text{Consumer } i \in \mathcal{I}(n) \text{ pays } \Lambda_n^* d_i^{0,*} + \frac{\lambda_n^*(\omega)}{\mathbf{P}\{\omega\}}\left(d_i^*(\omega) - d_i^{0,*}\right),$$
$$\text{Producer } j \in \mathcal{J}(n) \text{ is paid } \Lambda_n^* g_j^{0,*} + \frac{\lambda_n^*(\omega)}{\mathbf{P}\{\omega\}}\left(g_j^*(\omega) - g_j^{0,*}\right),$$

where recall that $\mathbf{P}\{\omega\}$ denotes the probability of scenario $\omega \in \Omega$.

[4]The authors in [13] model the cost of ramping from g_j^0 to $g_j(\omega)$ for each $j \in \mathcal{J}$, and d_i^0 to $d_i(\omega)$ for each $i \in \mathcal{I}$ in scenario $\omega \in \Omega$. We do not model such costs.

Next, we discuss the limitations of this design. First, notice that the authors in [13] propose a single-settlement market design. Their payments, however, bear a stark resemblance to that of a two-settlement one. Namely, the payment to each market participant consists of two parts. The first part defines a payment that treats $g^{0,*}$, $d^{0,*}$ as a forward dispatch. The second part then pays for deviation of $d^*(\omega), g^*(\omega)$ from said forward dispatch. It is unclear as to why one would design a mechanism that offers forward payments to consumers and renewable power producers, who unlike dispatchable power producers, do not have limits on their ramping capability.

Second, consider the merchandising surplus of the SO in scenario $\omega \in \Omega$, defined as the sum total of payments received from the consumers less the sum total of payments made to the producers in scenario ω. Denote this quantity by $\mathsf{MS}(\omega)$. The authors in [13] state and prove that $\mathbb{E}[\mathsf{MS}(\omega)] \geq 0$. In other words, their payment scheme is revenue adequate in expectation. However, for some realizations of ω, $\mathsf{MS}(\omega)$ can be negative. They provide an example to demonstrate the same. Lack of revenue adequacy leaves the SO with a negative cashflow in certain scenarios, making the market mechanism difficult to implement in practice.

V. Proposed Market Design

In this section, we present the proposed market design under uncertainty, and study its properties. In particular, we demonstrate that the market mechanism (1) results in an efficient dispatch plan, (2) is revenue adequate for all scenarios in Ω, and (3) supports a competitive equilibrium.

The market is run as follows.

- Given a collection of utility/cost functions for each consumer/producer at $t = 0$, the SO solves (4), and obtains d^*, $g^{0,*}$, g^*, and q^*.
- Let the optimal Lagrange multiplier for the power balance constraint in (4d) at node $n \in \mathcal{N}$ be $\lambda_n^*(\omega)$ in scenario $\omega \in \Omega$. Define
$$p_n^*(\omega) := \frac{\lambda_n^*(\omega)}{\mathbf{P}\{\omega\}}, \text{ for each } n \in \mathcal{N}, \omega \in \Omega. \qquad (8)$$
Then, $p^*(\omega)$ is the vector of nodal prices of power in scenario ω.
- To consumer $i \in \mathcal{I}(n)$, the SO reports d_i^* and p_n^*.
- To producer $j \in \mathcal{J}(n)$, the SO reports g_j^* and p_n^*.
- At $t = 1$, an $\omega \in \Omega$ is realized. The dispatch $d^*(\omega), g^*(\omega)$ is enforced.
- For each $n \in \mathcal{N}$, the payments in scenario $\omega \in \Omega$ are given by:
$$\text{Consumer } i \in \mathcal{I}(n) \text{ pays } p_n^*(\omega) d_i^*(\omega),$$
$$\text{Producer } j \in \mathcal{J}(n) \text{ is paid } p_n^*(\omega) g_j^*(\omega).$$

Remark 1. *In practice, the utility/cost functions used in solving (4) are constructed from demand bids/supply offers, submitted by the consumers/producers. In general, market participants may have incentives to misreport their utility/cost functions, i.e., they can behave strategically. Modeling and analyzing such strategic behavior in electricity markets is beyond the scope of the current paper, but stands as an interesting direction for future research.*

Recall that the proposed mechanism solves (4) with the utility/cost functions. Then, the computed dispatch plan is efficient, by definition. Next, we establish the revenue adequacy of the mechanism in Section V-A, and show that the outcome of the mechanism supports a competitive equilibrium in Section V-B.

A. Revenue adequacy

Recall that the merchandising surplus of the SO in scenario $\omega \in \Omega$, denoted by $\mathsf{MS}(\omega)$, is defined as the sum total of payments received from the consumers less the sum total of payments made to the producers in scenario ω. For the proposed mechanism, we have:

$$\mathsf{MS}(\omega) = \sum_{n\in\mathcal{N}} \sum_{\substack{i\in\mathcal{I}(n)\\ j\in\mathcal{J}(n)}} p_n^*(\omega)\left[d_i^*(\omega) - g_j^*(\omega)\right]. \tag{9}$$

Proposition 1. *Suppose d^*, $g^{0,*}$, g^*, and q^* solve the stochastic economic dispatch problem in (4), and p^* be defined as in (8). Then, the merchandising surplus of the system operator in each scenario $\omega \in \Omega$ satisfies*

$$\mathsf{MS}(\omega) = \sum_{n\in\mathcal{N}} \sum_{\substack{i\in\mathcal{I}(n)\\ j\in\mathcal{J}(n)}} p_n^*(\omega)\left[d_i^*(\omega) - g_j^*(\omega)\right] \geq 0. \tag{10}$$

The proposed mechanism is thus revenue adequate in each scenario $\omega \in \Omega$. We remark that the key step in the proof is in identifying $p^*(\omega)$ as the nodal prices of a deterministic economic dispatch problem[5], for which $d^*(\omega)$, $g^*(\omega)$ define the optimal dispatch.

B. Competitive equilibrium

Our goal here is to introduce a suitable notion of competitive equilibrium under uncertainty and demonstrate that the outcome of the proposed market design supports such an equilibrium. We begin by describing the *electricity market economy*, that comprises a collection of commodities, agents who trade in the commodities, their preferences, and the constraints on their consumptions or productions of said commodities. A competitive equilibrium then formalizes the outcome of trade among agents in such an economy. Power at each node in $\mathcal{N}$ and each scenario in Ω defines a separate commodity. There are three sets of agents in the economy: (1) the $|\mathcal{I}|$ consumers, each of whom demands power at a specific node in the power system in each scenario in Ω, (2) the $|\mathcal{J}|$ producers, each of whom produces power at a specific node in the power system in each scenario in Ω, and (3) the SO, who "buys" (possibly negative) amounts of power at each node and transports them across the power network, in each scenario in Ω.

In each scenario, (1) the payoff of a consumer is defined as the utility of consumption less his payment in that scenario, (2) a producer's payoff is given by his payment less the production cost in that scenario, and (3) the payoff of the SO, who buys power at each node (for transport), is given by the negative of the price at each node multiplied by the power injection at that node, summed across all nodes in the power network, in that scenario. The payoff of each agent is scenario-contingent. Assume all agents are *risk-neutral*. Then, at $t = 0$, each agent perceives his future contingent payoffs through its expectation. The expectation is calculated with respect to the distribution on Ω, he believes in. In light of Assumption 1, each agent holds the belief that the distribution on Ω is described by $\mathbf{P}$. At $t = 0$, each agent seeks to maximize his expected payoff (computed with respect to $\mathbf{P}$).

Consumer's problem: At $t = 0$, consumer $i \in \mathcal{I}(n)$ for $n \in \mathcal{N}$ chooses a consumption plan d_i, so as to maximize his total expected payoff $\mathbb{E}[u_i(d_i(\omega)) - p_n(\omega)d_i(\omega)]$, where $d_i(\omega)$ lies in the range of demands $[\underline{d}_i, \overline{d}_i]$. More formally, at $t = 0$, consumer $i \in \mathcal{I}(n)$ solves:

$$\begin{aligned} &\text{maximize} && \mathbb{E}[u_i(d_i(\omega)) - p_n(\omega)d_i(\omega)],\\ &\text{subject to} && \underline{d}_i \leq d_i(\omega) \leq \overline{d}_i,\\ &&& \text{for each } \omega \in \Omega, \end{aligned} \tag{11}$$

given the prices. He computes the expectation under the belief that $\mathbf{P}$ is the distribution over future states.

Producer's problem: For $j \in \mathcal{J}(n)$, producer j at $t = 0$ chooses (i) the generator set point g_j^0, and (ii) a production plan g_j, given the prices. Producer $j \in \mathcal{J}$ seeks to maximize his total expected payoff, given by $\mathbb{E}[p_n(\omega)g_j(\omega) - c_j(g_j(\omega))]$, within his capacity/ramping limits. More formally, producer $j \in \mathcal{J}$ at $t = 0$ solves:

$$\begin{aligned} &\text{maximize} && \mathbb{E}[p_n(\omega)g_j(\omega) - c_j(g_j(\omega))],\\ &\text{subject to} && \underline{g}_j(\omega) \leq g_j(\omega) \leq \overline{g}_j(\omega),\\ &&& |g_j^0 - g_j(\omega)| \leq \ell_j,\\ &&& \text{for each } \omega \in \Omega. \end{aligned} \tag{12}$$

System operator's problem: The SO buys power at each node $n \in \mathcal{N}$. Then, the SO chooses the $|\mathcal{N}|$-dimensional vector of nodal power injections $q(\omega)$ in scenario $\omega \in \Omega$. The SO seeks to maximize his expected payoff, which is given by $\mathbb{E}\left[\sum_{n\in\mathcal{N}} -p_n(\omega)q_n(\omega)\right]$, given the prices. Recall that the injection polytope $\mathcal{P}$ in (2) defines the region of feasible nodal power injections. Then, $q(\omega) \in \mathcal{P}$ for each $\omega \in \Omega$ constrains the SO's feasible trades. Formally, the SO solves:

$$\begin{aligned} &\text{maximize} && \mathbb{E}\left[\sum_{n\in\mathcal{N}} -p_n(\omega)q_n(\omega)\right],\\ &\text{subject to} && q(\omega) \in \mathcal{P},\\ &&& \text{for each } \omega \in \Omega. \end{aligned} \tag{13}$$

Equilibrium concept: An equilibrium in the electricity market economy would comprise consumption plans, generator set points, production plans, nodal power injection plans, and the conjectures of nodal prices of power in different scenarios. We present the definition of the competitive equilibrium for the electricity market economy under consideration.[6]

Definition 2 (Competitive equilibrium)**.** *A competitive equilibrium in the electricity market economy comprises of (1) the common price conjectures p^* of all consumers, producers, and*

[5] The stochastic economic dispatch problem, with $|\Omega| = 1$ defines a deterministic economic dispatch problem.

[6] The published version of this work somewhat erroneously calls this a Radner equilibrium.

the system operator, (2) consumption plans d^*, (3) generator set points $g^{0,*}$, (4) production plans g^*, and (5) nodal power injection plans q^*, that satisfies:

- *for each consumer $i \in \mathcal{I}(n)$, d_i^* solves (11), given that the prices at node n in scenarios in Ω will be given by p_n^*,*
- *for each producer $j \in \mathcal{J}(n)$, $g_j^{0,*}, g_j^*$ solve (12), given that the prices at node n in scenarios in Ω will be given by p_n^*,*
- *for the system operator, q^* solves (13), given that the prices across the network in scenarios in Ω will be given by p^*,*
- *the demand, the supply, and the injection of power are balanced at each $n \in \mathcal{N}$ and each $\omega \in \Omega$, i.e.,*

$$q_n^*(\omega) = \sum_{j \in \mathcal{J}(n)} g_j^*(\omega) - \sum_{i \in \mathcal{I}(n)} d_i^*(\omega).$$

for each $n \in \mathcal{N}$, $\omega \in \Omega$.

The relationship between competitive equilibrium and the proposed market mechanism is captured in the following result.

Proposition 2. *Suppose d^*, $g^{0,*}, g^*$, and q^* solve the stochastic economic dispatch problem in (4), and p^* be defined as in (8). Then, p^*, d^*, $g^{0,*}$, g^*, q^* constitute a competitive equilibrium in the electricity market economy.*

VI. An Illustrative Example

In this section, we study the proposed market design on a copperplate (single-bus) power system. Albeit stylized, the example reveals the explicit dependency of the dispatch and the payment scheme on the uncertainty of renewable supply. Consider a copperplate power system with four market participants: (1) an inelastic consumer D, (2) a flexible dispatchable generator (denoted as F) with infinite ramping capability, (3) an inflexible dispatchable generator (denoted as S) that cannot vary its output in different scenarios from its set point,[7] and (4) a wind power producer (denoted as W), that has no ramp limits. Define the uncertainty set $\Omega := [\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma] \subseteq \mathbf{R}_+$. Let $\mathbf{P}$ represent a uniform distribution on Ω.[8] The set Ω in this example is compact, and not finite as assumed in our model formulation. While this minor deviation does not introduce any conceptual difficulties, it has explanatory advantages. The power consumed by D is given by $\underline{d} = \overline{d} = d \geq \mu + \sqrt{3}\sigma$. Generator S produces $g_S(\omega) \geq 0$, incurring a cost $c_S \cdot g_S(\omega)$ for $c_S > 0$. Its ramping constraint is modeled by $\ell_S = 0$. Generator F produces $g_F(\omega) \geq 0$, incurring a cost $c_F \cdot g_F(\omega)$, where $c_F > c_S$. Its infinite ramping capability is modeled by letting $\ell_F = \infty$. The wind power producer W produces $g_W(\omega) \in [0, \omega]$, for $\omega \in \Omega$ at zero cost. The cost structures of F, S, and W represent a stylized version of electricity markets in practice in that (1) wind has negligible marginal costs of production, but is uncertain, (2) generators that are cheap, need to be dispatched forward, and (3) fast-ramping generators, capable of absorbing variability in real time, are costly.

In this example, the SO's problem (4) of maximizing the expected social welfare amounts to minimizing the expected total cost of production. Since $\ell_S = 0$, $g_S(\omega)$ equals $g_S \in \mathbf{R}$ for all scenarios $\omega \in \Omega$. For all variables of interest x, let x^* denote the corresponding variable at optimality in (4). We assume the cost of wind to be zero, and $c_F > c_S$. This implies

$$g_W^*(\omega) = \min\{d - g_S^*, \omega\}, \text{ and } g_F^*(\omega) = (d - g_S^* - \omega)^+,$$

where recall that $(x)^+ := \max\{x, 0\}$ for any $x \in \mathbf{R}$. Then, g_S^* minimizes $c_S g_S + c_F \mathbb{E}[(d - g_S - \omega)^+]$ over $[0, d - (\mu - \sqrt{3}\sigma)]$. Let $p^*(\omega)$ be the price of power, when the available wind is ω. It is easy to show that

$$p^*(\omega) := \begin{cases} c_F, & \text{if } g_F^*(\omega) \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

The next result characterizes the optimal production of S, the expected total cost of production under the proposed design, and the expected payments to various market participants.

Proposition 3. *The optimal production of S in all scenarios is given by:*

$$g_S^* = (d - \mu) + \left(1 - \frac{2c_S}{c_F}\right)\sqrt{3}\sigma,$$

and the expected total cost of production is given by:

$$\mathbb{E}\left[c_S g_S^* + c_F g_F^*(\omega)\right] = c_S\,(d - \mu) + c_S\left(1 - \frac{c_S}{c_F}\right)\sqrt{3}\sigma.$$

Moreover, the expected payments by D, and the expected payments to S,F and W are given by π_D, π_S, π_F, and π_W, respectively, where

$$\begin{aligned} \pi_D &:= c_S d, \\ \pi_S &:= c_S\,(d - \mu) + c_S\left(1 - 2\frac{c_S}{c_F}\right)\sqrt{3}\sigma, \\ \pi_F &:= \frac{c_S^2}{c_F}\sqrt{3}\sigma, \\ \pi_W &:= c_S \mu - c_S\left(1 - \frac{c_S}{c_F}\right)\sqrt{3}\sigma. \end{aligned}$$

The parameters σ and c_S/c_F, can have significant impact on the resulting dispatch and payments. The larger the σ, the greater the uncertainty in available wind production. The closer c_S/c_F is to unity, the lesser is the disparity between the costs of procuring power from S at $t = 0$ versus procuring power from F at $t = 1$. In short, σ is a measure of uncertainty, and c_F/c_S is a relative cost of 'waiting'. We study the effects of both these parameters on: (1) the optimal dispatch, (2) the difference in the expected total cost of production between the proposed design and the certainty equivalent dispatch, and (3) the distribution of payments between S, F, and W.

The optimal dispatch: As $\sigma \to 0$, the available wind becomes more certain, and $g_S^* \to d - \mu$. Also, $g_F^*(\omega) \to 0$, and $g_W^*(\omega) \to \mu$ for each $\omega \in \Omega$. As expected, the dispatch tends to the solution of (4), with no uncertainty in available wind.

[7] As an example, F can be a fast-ramping natural gas turbine, and S can be identified with a nuclear power plant.

[8] A uniform distribution on $[\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma]$ has mean μ and variance σ^2.

As $c_S/c_F \to 0$ (keeping c_F fixed), notice that $g_S^* \to d - (\mu - \sqrt{3}\sigma)$. Here, the cost of waiting is large, and all but the certain part of available wind, i.e., $\mu - \sqrt{3}\sigma$, tends to be procured from S. And, F tends to produce less. Similarly, as $c_S/c_F \to 1$ (keeping c_F fixed), $g_S^* \to d - (\mu + \sqrt{3}\sigma)$. In this case, the cost of waiting is small, and hence, the optimal dispatch tends to compensate for lack of available wind by utilizing F.

Expected total cost of production: The expression derived for the expected total cost of production can be split into two parts: (i) the contribution from the mean of the available wind, i.e., $c_S\,(d-\mu)$, and (ii) the component that depends on the variability of the wind, i.e., $c_S\left(1 - \frac{c_S}{c_F}\right)\sqrt{3}\sigma$. The second component increases linearly with σ. As $c_S/c_F \to 1$, the second component approaches zero. This is a consequence of the symmetry of the distribution of available wind around μ.

Under the certainty equivalent dispatch, it is easy to verify that S produces $\hat{g}_S := d - \mu$, and F produces $\hat{g}_F(\omega) = (\mu - \omega)^+$, when the available wind is ω. The expected total cost of production is then given by:

$$\mathbb{E}\left[c_S \hat{g}_S + c_F \hat{g}_F(\omega)\right] = c_S(d-\mu) + \frac{c_F}{4}\sqrt{3}\sigma,$$

implying

$$\begin{aligned}&\mathbb{E}\left[c_S \hat{g}_S + c_F \hat{g}_F(\omega)\right] - \mathbb{E}\left[c_S g_S^* + c_F g_F^*(\omega)\right] \\ &\quad = \frac{c_F}{4}\left(1 - 2\frac{c_S}{c_F}\right)^2 \sqrt{3}\sigma.\end{aligned}$$

The expected total cost of production is lower in our proposed design than with the certainty equivalent dispatch. The difference in the expected total costs grows linearly with σ, revealing that the suboptimality of the certainty equivalent dispatch grows with the variability in renewable supply.

Distribution of payments among the market participants: As σ increases from 0, the wind becomes more variable, and the resulting payment to W, i.e., π_W, decreases. The reduction in π_W is distributed among the expected payments to S and F. While π_S may increase/decrease depending on the ratio c_S/c_F, the payment π_F always increases. In this example, the mean of the available wind, i.e., μ, has no bearing on π_F. In a sense, the fast-ramping dispatchable resource F derives its value purely from the variability of the free but uncertain wind power production.

Competitive equilibrium: In Section V-B, we argued that the proposed market mechanism supports a competitive equilibrium. We discuss the equilibrium within the context of this example. Recall that $p^*(\omega)$ in (14) defines the price of power in scenario $\omega \in \Omega$. Using g_S^* from Proposition 3, we have:

$$p^*(\omega) = \begin{cases} c_F, & \text{if } \omega \in \left[\mu - \sqrt{3}\sigma, \mu - \sqrt{3}\sigma + \frac{2c_S}{c_F}\sqrt{3}\sigma\right], \\ 0, & \text{otherwise.}\end{cases} \tag{15}$$

Consider how S approaches its production plan, given the prices. Being a producer lacking ramping abilities, S seeks $g_S \geq 0$ that maximizes his expected profit $\mathbb{E}[p^*(\omega)g_S - c_S g_S]$. The above statement is derived under three assumptions: (i) S is risk-neutral, and hence, perceives the random profits at $t = 1$ through its expectation, (ii) S is price-taking, and hence, does not assume that his choice of g_S influences the prices at $t = 1$, (iii) S holds the belief that $\mathbf{P}$ describes the probability distribution on Ω (Assumption 1), and hence, computes the expectation with respect to $\mathbf{P}$. Thus, S conjectures $\mathbb{E}[p^*(\omega)] = c_S$. In turn, his expected profit is zero under said price conjectures, making S agnostic to his choice of the set point. Then, g_S^*, defined in Proposition 3, defines an optimal choice for S at $t = 0$. At $t = 1$, ω is realized. Without the flexibility of ramping, S produces at the said set point. Similar conclusions can be drawn regarding the choices g_F^* and g_W^* by F and W, respectively, when the power prices in different scenarios are given by (15).

VII. Comparison with Existing Designs

The goal of this section is to contrast the proposed market design in Section V with the two existing designs, introduced in Section IV. We compare the designs, both with respect to the properties of the market mechanism and the practical considerations necessary for its implementation.

A. *Efficiency and revenue-adequacy*

Consider the certainty equivalent dispatch in Section IV-A. We have already argued that such a scheme will not, in general, yield an efficient dispatch plan. The payment scheme accompanying the certainty equivalent dispatch, however, can be shown to be revenue adequate in all scenarios. Next, consider the market mechanism in Section IV-B, that is adopted from [13]. The dispatch scheme in (7) is the SED in (4) with additional constraints involving the generator set-points. One can conceive examples, where g^*, d^*, q^*, that are optimal in (7), are not optimal in (4). As a result, the dispatch plan may not be efficient. Moreover, the merchandising surplus for the SO in certain scenarios can be negative. The proposed mechanism, however, addresses both of these concerns. Specifically, it results in an efficient dispatch plan, and the SO is revenue adequate in all scenarios.

B. *Communication requirements*

Consider the certainty equivalent dispatch and its payment scheme in Section IV-A. The SO solves (5) at $t = 0$, and solves (6) at $t = 1$. Each market participant receives four parameters from the SO, that define his consumption/production and payment. For example, consumer $i \in \mathcal{I}(n)$ receives D_i^*, Λ_n^*, $d_i^*(\omega)$ and $\lambda_n^*(\omega)$, as defined in Section IV-A. One can verify that the design in Section IV-B also requires the SO to communicate four numbers to each market participant.

On the contrary, the market proposal in this paper requires the SO to communicate the consumption/production/pricing plans to each market participant. Under the hypothesis Ω is discrete, the SO needs to report $2|\Omega|$ parameters to each consumer/producer. The number of representative scenarios of available renewable supply grows with the number of renewable power producers in the network, leading to a communicational burden on the SO. It remains to be seen whether such burdens are prohibitive in practice.

C. Volatility in realized payments

The payment to each market participant in the existing designs, described in Section IV, has two parts to the payment – a forward component defined at $t = 0$, and another that depends on the scenario realized at $t = 1$. The proposed design, on the other hand, has no forward component. One can demonstrate by designing suitable examples that the realized payments in the proposed design can exhibit higher volatility than the realized payments in the existing designs. Such volatility does not affect a risk-neutral market participant. However, electricity market participants are risk-averse in practice [17], [18]. Hence, consumers/producers bear larger financial risks under the proposed design. We envision designing a market for financial assets, that is simultaneously run along with the day-ahead energy market, where market participants can hedge such financial risks. We remark that for a suitably designed asset market, the risk aversion only affects the asset prices and the portfolios held by the market participants; the dispatch and the payments in the electricity market would remain unaltered. The details of such a concomitant asset market design is left for future work.

VIII. Conclusions

In this paper, we propose a novel market mechanism for electricity markets under uncertainty. Our single settlement market design (1) results in an efficient dispatch, (2) is revenue adequate in all scenarios of available renewable supply, and (3) supports a competitive equilibrium. The mechanism compensates for the shortcomings of existing designs in the literature. However, it entails a paradigm in which the system operator must communicate a dispatch and a pricing plan to each market participant. That significantly increases the complexity of market operations.

There are a number of possible future directions to explore. First, we have argued that the resulting payments to market participants under the proposed mechanism are more volatile than in existing designs. An important question is one of designing financial instruments to hedge such risks and study the effect of risk aversion on the prices of such instruments. Second, our equilibrium analysis hinges on the assumption that consumers/producers in the wholesale electricity markets are price-taking. However, there is evidence [19], [20] that such markets are susceptible to market manipulation. Though the precise mathematical formulations vary, authors in [21]–[25] illustrate the role of multiple settlements in reducing the impact of strategic interaction of market participants. Therefore, we wish to build on our single-settlement design to define a two-settlement one, and further characterize the effect of strategic interaction by analyzing it in a game theoretic framework.

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