Business-to-Peer Carsharing Systems with Electric Vehicles

Theodoros Mamalis Subhonmesh Bose Lav R. Varshney

***Abstract*—Carsharing systems are adopting electric vehicles into their fleets and may therefore be able to provide not only transportation services to drivers/passengers but also energy services to the power grid through appropriate participation in electricity markets. This paper develops a queuing model of such carsharing platforms where cars may be deployed for transportation services at a given price, but also for grid services during transportation-idle periods through energy price arbitrage. The model provides a characterization of revenue from each of these two streams. The paper further finds optimal pricing and battery splitting to maximize revenue for the carsharing platform, via an analysis of the reward structure and an optimization algorithm. Platform revenue is assessed for various system parameters under optimal operation.**

***Index Terms*—Electric vehicles, queuing networks, sharing economy, transportation, revenue maximization**

I. Introduction

Automobiles have a certain flexibilty in that they can act as both transportation resources and as distributed energy resources. This dual-use flexibility of gasoline engines was exploited in the early twentieth century by car owners at the individual level, e.g. through Model T-powered washing machines [1, Fig. 2]. The resurgence of modern plug-in electric vehicles (EVs), however, enables leveraging this flexibility at a large-scale systems level through coupled control of transportation systems and energy grids [2], especially in the context of the sharing economy [3]. Indeed on the energy side, EVs can not only serve as both energy sources and sinks, but also provide voltage and frequency regulation services to the smart grid [4]–[7]. On the transportation side, business-to-peer carsharing platforms[1] are starting to adopt electric vehicles within their fleets [8]–[10]. This is especially the case for automobile manufacturer-based platforms including Volkswagen's WE, Mercedes-Benz's Car2Go, BMW's ReachNow, GM's Maven, and Audi's Audi-on-Demand.

Carsharing companies can garner additional revenues from utilizing the battery packs of their vehicle fleet to provide grid services. This paper develops a rigorous queuing-theoretic model and computational tools to facilitate the analysis and optimal coordination of EV-based carsharing service platforms. Although queuing models have a long history in modeling transportation services [11]–[13], including carsharing platforms [14], we believe this is the first application to settings with EVs. The main goal in developing our queuing-theoretic model for carsharing platforms is to (i) capture the salient features of EV charging processes, and (ii) capture the tradeoffs in providing both transportation and grid services. Indeed, the inherent time required to charge EV batteries will impact a transportation service provider's ability to deliver rides in a timely manner. Further, the underlying cost and time requirements of battery charging will affect the pricing decisions for mobility services.

In particular, we first establish a simple queuing model of EV transportation service provision and recharge, further determining the revenue garnered. Next we consider the possibility of using an EV battery to perform price arbitrage in an energy market; this is done in the presence of a stochastic real-time wholesale market price signal through a dynamic programming argument. Finally, we consider the possibility of (dynamically) splitting the battery into a transportation segment and a grid services segment to enable energy price arbitrage when idle from delivering transportation services. For certain standard distributional assumptions, we characterize the candidate optimal transportation prices p upon fixing the battery capacity B dedicated to provide transportation services. An efficient algorithm for joint optimization of (p, B) is given, and numerical examples are used to provide insight into the basic tradeoff.

For ease of presentation and to capture the basic queuing-theoretic insights, we focus on energy services restricted to price arbitrage against real-time prices in the energy market. We further focus on the setting where the fleet consists of a single car. Longer presentations of this work will include the setting with a variety of grid services and larger fleets of cars.

II. Modeling transportation services

Let customers open $\mathbf{A}$'s application following a Poisson process with intensity λ_0. When a customer arrives, she observes the posted price p (in money/time) for utilizing $\mathbf{A}$'s vehicle and the maximum driving time $\tau^{\max} = B/\beta^-$, assuming that the vehicle battery of capacity B depletes at a constant rate β^- when driven. She decides to rent a vehicle, if (i) the posted price p is lower than her reservation price, and (ii) her required driving time τ does not exceed the maximum driving time $\tau^{\max}$. We model a customer's reservation price and driving time as independent random variables. Let $\overline{F}_\pi$ denote the complementary cumulative or tail distribution function of the reservation wages, and F_τ denote the cumulative distribution function of the trip times. It follows that customers who are willing to pay the posted price and

All authors are with the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA. Emails: {mamalis2, boses, varshney}@illinois.edu.

Discussions with Tamer Başar are appreciated. This work was supported in part by a grant from the Siebel Energy Institute, and by the Andreas Mentzelopoulos Scholarships for the University of Patras.

[1]Business-to-peer carsharing platforms provide short-term (e.g., hourly) car rentals, where the customer drives the car provided by the company. On the other hand peer-to-peer ride sharing platforms, such as Lyft and Uber, rely on privately-owned vehicles and their drivers to provide on-demand mobility to customers.

abide by the driving time restrictions arrive according to a Poisson process with intensity

$$\lambda := \lambda_0 \overline{F}_\pi(p) F_\tau(B/\beta^-). \quad (1)$$

Driving times of customers who ultimately avail $\mathbf{A}$'s service follow the same distribution as the driving times of all customers truncated at B/β^-. The vehicle battery loses $\beta^-\tau$ amount of energy when driven for τ, where recall that β^- denotes the battery discharge rate. Denote the battery charging rate at the depot by β^+. Then, the car will require $\tau_C := \frac{\beta^-}{\beta^+}\tau$ amount of time to charge it back up to its capacity B. Therefore, we model the charging time proportional to the driving time as Figure 1(a) illustrates. The car (server) is deemed busy when it is either being driven or charging after being driven. For each ride provided, the car remains busy for $\tau + \tau_C = \tau(1+\beta^-/\beta^+) := \tau\tilde{\beta}$ amount of time, implying that the service time follows a truncated version of the trip time distribution with mean

$$\frac{1}{\mu} := \tilde{\beta}\mathbb{E}[\tau \mid \tau \leq B/\beta^-]. \quad (2)$$

Thus, $\mathbf{A}$'s service has been modeled as an $M/G/1$ queue, written in Kendall's notation, with arrival rate λ and service rate μ. In our current formulation, we allow each arriving customer to wait in the depot if the vehicle has been checked out by another customer. A more realistic model with reneging or balking, and other behavioral models of customers as in [15] are left to future endeavors.

Too low a price or too large a battery capacity can result in unstable growth in the passenger queue. Enforcing $\lambda < \mu$ ensures that the queue remains stable, i.e.,

$$\lambda_0 \tilde{\beta} \overline{F}_\pi(p) \underbrace{\int_0^{B/\beta^-} t f_\tau(t) dt}_{:=\Phi_\tau(B/\beta^-)} < 1, \quad (3)$$

where f_τ is the probability density function of driving times.

A. *Computing* $\mathbf{A}$*'s revenue rate from transport services*

Drawing results from the equilibrium analysis of $M/G/1$ queues, we aim to calculate the rate at which $\mathbf{A}$ accrues revenue, assuming the car service is in steady-state. From renewal-reward theory, this rate is given by the ratio of the expected revenue $\mathbf{A}$ makes from a busy period and the expected total cycle length (busy + idle period). We calculate the expected revenue during a busy period as follows. For each customer, $\mathbf{A}$ is paid $p\tau$ for the transportation service, and it pays $p_{\mathsf{ret}}\beta^+\tau_C = p_{\mathsf{ret}}\beta^-\tau$ for energy. Here, p_{ret} (in money/energy) denotes the flat retail rate for energy that the distribution utility (or a retail aggregator) charges $\mathbf{A}$. Overall, $\mathbf{A}$ makes $(p - p_{\mathsf{ret}}\beta^-)\tau$ from a customer who drives it for τ time. Thus, the expected revenue from the customers in a busy period from transportation service provision is given by

$$\Pi_T = \mathbb{E}\left[\sum_{i=1}^{N_T}(p - p_{\mathsf{ret}}\beta^-)\tau_i\right],$$

where $\tau_1, \ldots, \tau_{N_T}$ are the driving times of the N_T customers in the busy period who seek to use the car service. By Wald's lemma, the above relation simplifies to

$$\begin{aligned}\Pi_T &= (p - p_{\mathsf{ret}}\beta^-)\mathbb{E}[N_T] \cdot \mathbb{E}[\tau \mid \tau \leq B/\beta^-] \\ &= (p - p_{\mathsf{ret}}\beta^-)\frac{\lambda}{\mu - \lambda}\mathbb{E}[\tau \mid \tau \leq B/\beta^-].\end{aligned}$$

The last line in the above equation uses the fact that the expected length of the busy period for an $M/G/1$ queue is given by $(\mu-\lambda)^{-1}$. Also, the expected length of the idle period is λ^{-1}, yielding the following expression for the reward rate R_T of $\mathbf{A}$ from transportation service provision.

$$\begin{aligned}R_T &= \left(\frac{1}{\mu - \lambda} + \frac{1}{\lambda}\right)^{-1}\Pi_T \\ &= (p - p_{\mathsf{ret}}\beta^-)\frac{\lambda^2}{\mu}\mathbb{E}[\tau \mid \tau \leq B/\beta^-].\end{aligned}$$

Utilizing (1) and (2), we get

$$R_T = \lambda_0^2 \tilde{\beta}(p - p_{\mathsf{ret}}\beta^-)\bar{F}_\pi^2(p)\Phi_\tau^2(B/\beta^-). \quad (4)$$

Notice that we do not explicitly model $\mathbf{A}$'s maintenance and repair costs for its vehicles. One can expect repair costs to be proportional to the driving times, as more time a car is driven, the more prone it becomes to traffic accidents. Such costs can be included in the retail electricity price. Regular maintenance costs will add a constant to the revenue rate that will not affect our conclusions from the model.

III. Price arbitrage using vehicle battery during idle periods

We aim to find how a vehicle can split its battery capacity for dual use—a portion set aside to provide transportation services and utilize the rest to maximize its revenue from arbitraging against time-varying energy prices. To simplify the exposition, conceptually consider a vehicle battery with capacity B_{tot} as a combination of a transportation battery of capacity B and a trading battery of capacity $B' = B_{\text{tot}} - B$.

Conceptually splitting the vehicle battery into two parts ensures that a transportation customer always enjoys a nonrandom battery capacity for her trip. Allowing the vehicle battery to provide grid services with the total battery capacity makes the available battery for transportation a random quantity, compromising $\mathbf{A}$'s carsharing business. Any residual energy in the grid battery can be utilized by a transportation customer in an emergency. Further, the battery split need not be constant, but can be varied over time. For example, on days with high customer traffic, $\mathbf{A}$ might allocate a higher portion of the battery for transportation as opposed to that in a day with low traffic.

Recall that the car is busy with transportation service when it is being driven by a customer, or is preparing for it by charging its transportation battery. It is idle, otherwise. During this idle period, let $\mathbf{A}$ receive nonnegative[2] energy prices ρ (in money/capacity) at regular intervals of length Δ as shown in

[2]The nonnegativity assumption on the prices can be relaxed with minor modifications to the results.

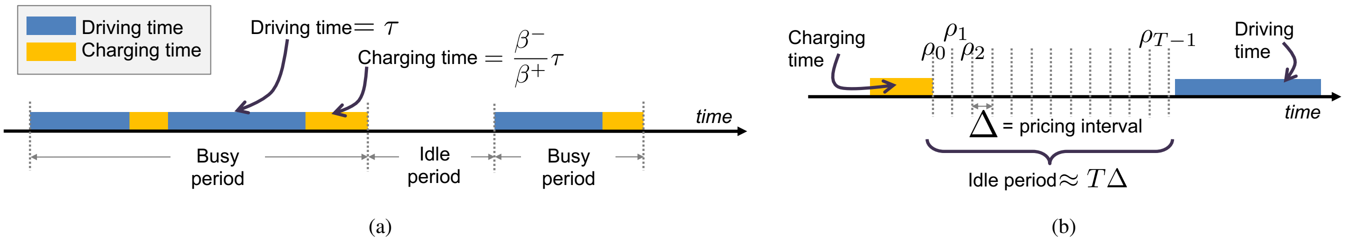

Fig. 1: (a) The relation between driving times, charging times, and busy times. (b) The price process during the idle period.

Figure 1(b). These prices may be the locational marginal prices from a real-time wholesale market,[3] or from an emergent retail market. Notice that ρ is different from the flat retail energy price p_{ret} that $\mathbf{A}$ pays to charge its transportation battery. $\mathbf{A}$ participates as a regular consumer of a distribution utility that offers it a flat retail rate to charge its transportation battery. And during an idle period, $\mathbf{A}$ participates as an owner-operator of a distributed energy resource (the vehicle battery) or one who relinquishes control over its trading battery to an aggregator of such resources.

A vehicle is connected to the grid when it charges its transportation battery. We insist in our model that it does not arbitrage energy to accrue revenue during that period. That is, we separate the times when the car provides or prepares for transportation services, and when it takes part in price arbitrage. This separation facilitates easy auditing.

A. Optimal control for price arbitrage

We now formulate the question of expected revenue maximization from price arbitrage with a battery of capacity B' over an idle period as a discrete-time stochastic control problem. Suppose there are T intervals of length Δ within an idle period, where recall that Δ is the time interval between consecutive price changes. Let $\boldsymbol{\rho} := (\rho_0, \ldots, \rho_{T-1})$ denote the stochastic price process against which $\mathbf{A}$ maximizes its expected revenue from arbitrage. Starting from a state of charge $z_0 \in [0, B']$, the trading battery state at interval t progresses as

$$z_{t+1} := z_t + u_t$$

for $t = 0, \ldots, T-1$. Here, u_t stands for the energy transacted at time t. A positive u indicates charging the trading battery, and a negative one indicates discharging.

We seek a control policy $\boldsymbol{\gamma} := \{\gamma_0, \ldots, \gamma_{T-1}\}$, where γ_t maps the available information at time t to the storage control action u_t^{γ}. The relevant information for control design comprises the state at that time and the history of prices till that time. A policy is deemed admissible, denoted $\boldsymbol{\gamma} \in \mathcal{M}(B')$, if the induced control actions respect the capacity constraints of the trading battery, i.e.,

$$0 \le z_0 + \sum_{k=0}^{t} u_k^{\gamma} \le B'$$

almost surely for $t = 0, \ldots, T-1$. The optimal expected revenue over an idle period of length T is then given by

$$J^*(z_0) := \max_{\boldsymbol{\gamma} \in \mathcal{M}(B')} \mathbb{E}\left[\sum_{k=0}^{T-1} \rho_k(-u_k^{\gamma})\right], \qquad (5)$$

where $\mathbb{E}$ stands for the expectation computed with respect to the distribution on the prices. In our next result, we provide a closed-form expression for the optimal revenue $J^*(z_0)$. The proof in the Appendix relies on a dynamic programming based argument, adopted from [16]. We use the notation $z^+ = \max\{z, 0\}$ in stating the result.

Proposition 1. *The maximum expected revenue from price arbitrage with a trading battery of capacity B' is given by*

$$J^*(z_0) = z_0\mathbb{E}[\rho_0] + B' \sum_{j=0}^{T-2} \mathbb{E}\left[\left(\rho_{j+1|j} - \rho_j\right)^+\right], \qquad (6)$$

where $\rho_{t+1|t} = \mathbb{E}\left[\rho_{t+1}|\rho_{\le t}\right]$ is the one-step look-ahead price forecast, given the history of prices till time t.

The derivation of the above result proves that the optimal storage control policy has a threshold structure. It prescribes to charge the trading battery completely when the price is expected to go up in the next time step, and to fully discharge it, otherwise.

Our model of storage operation for price arbitrage neglects three important considerations. First, we do not model roundtrip efficiency losses. The results can be extended to consider such losses, but are not modeled to maintain clarity of exposition. Second, we do not model the effect of battery degradation from storage cycling. Vehicle batteries have limited cycle life and replacement costs can be significant, e.g., see [17]. We aim to address this modeling limitation in future work. Finally, we do not consider ramping limitations on the battery's charging and discharging abilities. One way to account for ramping limitations is to constrain the split of the battery in a way that the trading battery size is chargeable within Δ time.

[3] While $\mathbf{A}$ needs a sufficiently large aggregate battery capacity to participate in a wholesale market, our analysis on the utilization of one car battery reveals the dependency of the revenue rate on said battery capacity and informs future work with multiple vehicles.

B. Computing $\mathbf{A}$'s revenue rate from price arbitrage

With the aid of Proposition 1, we now derive the revenue rate from energy trading that $\mathbf{A}$ garners. To simplify the calculation, assume that the price process $\boldsymbol{\rho}$ is stationary Markov[4]. Then, the distribution of $\tilde{\rho}_t := \left(\rho_{t+1|t} - \rho_t\right)^+$ becomes independent of t. Denoting its expectation by $\langle\tilde{\rho}\rangle$, the expected revenue during an idle period becomes

$$\Pi_E \approx B' \cdot \mathbb{E}[T-2]\langle\tilde{\rho}\rangle \approx \frac{B'}{\lambda\Delta}\langle\tilde{\rho}\rangle,$$

where Δ denotes the length of the pricing interval. We make two approximations in deriving Π_E. First, we ignore the contribution of the initial state of charge at the start of an idle period to the revenue from arbitrage. Second, the number of pricing intervals $T-2$ has been approximated by T whose expectation is given by $(\lambda\Delta)^{-1}$. If the number of pricing intervals are sufficiently high within an idle period, these approximations are accurate.

The expected revenue in the above relation yields the following revenue rate from energy trading using renewal-reward theory.

$$\begin{aligned} R_E &= \left(\frac{1}{\mu-\lambda} + \frac{1}{\lambda}\right)^{-1} \Pi_E \\ &= \frac{1}{\Delta} B' \langle\tilde{\rho}\rangle \left[1 - \lambda_0 \tilde{\beta} \bar{F}_\pi(p) \Phi_\tau(B/\beta^-)\right]. \end{aligned}$$

When splitting the vehicle battery for transportation and price arbitrage, a larger trading battery size increases the arbitrage revenue in each idle period. Also, it leaves lesser capacity for transportation, leading to a decrease in the incoming traffic of customers seeking transport, thereby increasing the idle time. The transportation price also has a similar effect on the revenue rate from energy trading in that it impacts the rate of arriving customers that in turn affects the lengths of the idle periods.

[4]The published version is missing the Markov assumption.

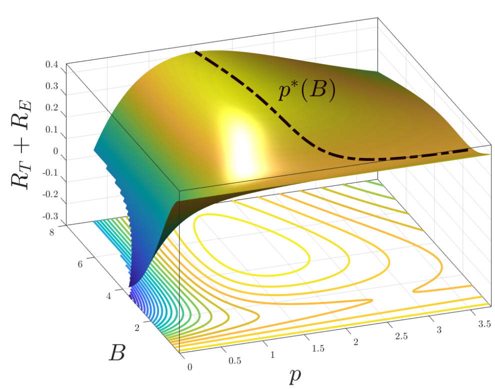

Fig. 2: Variation of aggregate revenue rate with transportation price and battery allocated for transportation. For this experiment, we chose $\lambda_0 = 1$, $B_{\text{tot}} = 8$, $\beta^- = 1$, $\beta^+ = 7$, $p_{\text{ret}} = \frac{3}{10}$, and $\frac{\langle\tilde{\rho}\rangle}{\Delta} = \frac{1}{20}$. Trip times and reservation wages were assumed to be exponentially distributed with means 1, and $\frac{5}{2}$, respectively. The dashed line plots $p^*(B)$ in (7) for each $B \in [B^{\mathsf{L}}, B_{\text{tot}}]$.

IV. Splitting Battery Capacity for Dual Use

Having computed the revenue rate from transportation and energy trading for price arbitrage, we now derive how $\mathbf{A}$ can split its battery capacity to maximize its aggregate revenue rate $R_{\text{tot}} := R_T + R_E$. Recall that we concluded Section III with the observation that the transportation price p affects not only the revenue from transportation, but also the revenue from price arbitrage. Hence, $\mathbf{A}$ seeks to jointly optimize its transportation price and the battery split for the two services.

$$\begin{aligned} \underset{p,B}{\text{maximize}} \quad & \lambda_0^2 \tilde{\beta}(p - p_{\text{ret}}\beta^-) \bar{F}_\pi^2(p) \Phi_\tau^2(B/\beta^-) \\ & + \frac{\langle\tilde{\rho}\rangle}{\Delta}(B_{\text{tot}} - B)\left[1 - \lambda_0\tilde{\beta}\bar{F}_\pi(p)\Phi_\tau(B/\beta^-)\right], \\ \text{subject to} \quad & \lambda_0 \tilde{\beta} \bar{F}_\pi(p) \Phi_\tau(B/\beta^-) < 1, \\ & p \geq 0, \ 0 \leq B \leq B_{\text{tot}}. \end{aligned}$$

The stability constraint defines an open set. To avoid technical difficulties in optimizing over such an open set, we take its closure with the understanding that the parameters are suitably perturbed to make the closed set feasible.

Optimizing the aggregate revenue rate R_{tot} can be challenging, owing to its nonlinear non-concave variation in the parameters p and B as Figure 2 illustrates. In what follows, we make additional assumptions on the trip times and identify structural properties of R_{tot} that facilitate the development of an algorithm towards solving the above optimization problem.

A. The case with exponentially distributed reservation prices

We characterize the candidate optimal transportation prices upon fixing the battery capacity B dedicated to provide transportation services.

Proposition 2. *Suppose the reservation prices for transportation customers are exponentially distributed with mean $\langle\pi\rangle$. For a given $B \in (0, B_{tot}]$, the maximum of R_{tot} occurs at $p^* = +\infty$ or at*

$$p^*(B) = \max\left\{p_0 - \langle\pi\rangle\mathcal{W}\left(-\frac{p_1}{\langle\pi\rangle}e^{\frac{p_0}{\langle\pi\rangle}}\right), \ p_{stable}\right\}, \quad (7)$$

if $1 + p_0/\langle\pi\rangle + \log(p_1/\langle\pi\rangle) \geq 0$, where $\mathcal{W}$ is the principal branch of the Lambert-W function, and

$$p_0 := p_{\text{ret}}\beta^- + \frac{1}{2}\langle\pi\rangle, \quad p_1 := \frac{(B_{tot} - B)\langle\tilde{\rho}\rangle}{2\lambda_0\Delta\Phi_\tau(B/\beta^-)},$$
$$p_{stable} := \bar{F}^{-1}\left([\lambda_0\tilde{\beta}\Phi_\tau(B/\beta^-)]^{-1}\right).$$

Proposition 2 is crucial to our design of an algorithm to compute the optimal battery split B^* and the optimal price p^*. Several remarks on the result are in order before presenting its proof and the algorithm design. The ensuing discussion ignores the stability constraints for the ease of exposition.

First, the Lambert-W function is a solution to the implicit equation $\mathcal{W}(xe^x) = x$. For $x \in [-1/e, 0)$, there are two solutions to that equation, both of which are negative. The principal branch selects the one with the smallest absolute value; see [18] for details. As a result, the candidate optimizer $p^*(B)$ in (7) is no less than p_0.

Second, notice that as B sweeps from the total battery capacity to zero, p_1 increases from zero to ∞. Said differently, $\mathbf{A}$ charges its passengers more as the size of the trading battery $B_{tot} - B$ increases, to both compensate from reduced passenger car-requests and to garner higher energy-trading revenue from increased idle times. Further, beyond a certain size of the trading battery, p_1 becomes large enough to make the argument inside the Lambert-W function less than $-1/e$. Then $p^*(B)$ in (7) no longer defines a candidate optimizer. In other words, when the trading battery is large enough, marginally increasing the trading battery will marginally increase the price-arbitrage profit, but this increase is unable to cover $\mathbf{A}$'s subsequent marginal loss in transportation revenue, and $p^* = \infty$ becomes the only candidate optimizer for such B values. Therefore, $p^*(B)$ in (7) is a candidate optimizer only for $B \in [B^{\mathsf{L}}, B_{\text{tot}}]$, where B^{L} is implicitly defined by

$$B^{\mathsf{L}} + \frac{1}{\langle \tilde{\rho} \rangle} 2\lambda_0 \Delta \langle \pi \rangle e^{-(1+p_0/\langle \pi \rangle)} \phi_\tau (B^{\mathsf{L}}/\beta^-) = B_{\text{tot}}. \quad (8)$$

Third, $p^* = \infty$ corresponds to the case that no customer avails $\mathbf{A}$'s transportation service, effectively reducing its vehicle to a static battery. It readily follows that $\mathbf{A}$ will simultaneously reduce B to zero, thus maximizing its revenue from energy trading.

Fourth, setting $B = B_{\text{tot}}$ amounts to fully utilizing the vehicle battery for transportation. In that case, the only candidate optimal transportation price becomes p_0 that depends both on the retail energy price and the mean reservation price. Higher the energy retail price to charge its vehicles, the more $\mathbf{A}$ charges customers to compensate. Higher the customers' mean reservation price, the more $\mathbf{A}$ charges them to exploit it.

Proof of Proposition 2. We ignore the queue stability constraint in the rest of the proof. The derivative of R_{tot} with respect to p is given by

$$\frac{\partial R_{\text{tot}}}{\partial p} = \lambda_0 \tilde{\beta} \Phi_\tau (B/\beta^-) \bar{F}_\pi(p) f_\pi(p) \left(p_1 e^{\frac{p}{\langle \pi \rangle}} + p_0 - p \right),$$

where recall that f_π is the probability density function of the reservation prices. Therefore, the derivative is positive at zero and for large p. The candidate maximizers of the aggregate revenue are infinity and the roots of the derivative. These roots are the solutions of $p_1 e^{\frac{p}{\langle \pi \rangle}} + p_0 - p = 0$, which can be written as

$$\frac{1}{\langle \pi \rangle}(p_0 - p) e^{\frac{1}{\langle \pi \rangle}(p_0 - p)} = -\frac{p_1}{\langle \pi \rangle} e^{\frac{p_0}{\langle \pi \rangle}}.$$

Applying $\mathcal{W}$ on both sides, we infer that the above relation has at most two solutions. A candidate maximizer of R_{tot} is the smallest root, given by (7). Further, $\mathcal{W}$ is only well-defined over negative arguments in $[-1/e, 0)$, leveraging which, the inequality identifying a potential optimizer in (7) follows. ■

B. Algorithm to optimize transportation price and battery split

Proposition 2 allows us to design the following algorithm to optimize the transportation price and the battery split for dual use. For convenience, we use the notation $R_{\text{tot}}(p, B)$ to make explicit the dependency of the aggregate revenue rate on p and B.

- Compute B^{L} in (8) using a bisection search over $[0, B_{\text{tot}}]$.
- Maximize $R_{\text{tot}}(p^*(B), B)$ over $[B^{\mathsf{L}}, B_{\text{tot}}]$ using projected gradient descent with diminishing step sizes, starting from $(B^{\mathsf{L}} + B_{\text{tot}})/2$. Call the optimizers $p^{*,1}, B^{*,1}$.
- If $R_{\text{tot}}(\infty, 0) > R_{\text{tot}}(p^{*,1}, B^{*,1})$, then return $p^* = \infty, B^* = 0$. Otherwise, return $p^{*,1}, B^{*,1}$.

We cannot guarantee that the gradient method converges to the true optimizer in general. It does so for the numerical experiments described next.

C. Variation of optimal solution with problem parameters

Figure 3 illustrates the variation of the optimal revenue rate R^*_{tot} and the transportation battery capacity B^* at optimality with various problem parameters. As one expects, $\mathbf{A}$ invests all its battery for grid services for low customer traffic (low λ_0). The part dedicated to transportation increases with the customer traffic upto a point, after which transportation becomes more attractive than grid services. Similarly, the longer the expected trip times $\langle \tau \rangle$ the higher the revenue the platform accrues from transportation. Therefore, when trip times become high enough the battery is allocated fully to carsharing services. Next, recall that $\langle \tilde{\rho} \rangle / \Delta$ equals the expected gain from energy arbitrage with unit battery capacity, and its effect on B^* is exactly opposite to that of λ_0 and $\langle \tau \rangle$. As $\langle \tilde{\rho} \rangle / \Delta$ increases, grid services become more rewarding, and thus our algorithm favors a battery split against transportation services, reducing it to zero when the grid profits become high enough.

V. Conclusions and Future Research Directions

This paper defines a framework to analyze carsharing services that can utilize the vehicle battery for grid services when idle. The framework conceptually splits the battery capacity into two parts—one solely for transportation services and the other for grid services. To keep the analysis concrete, the focus is on using the battery for price arbitrage against real-time prices during $\mathbf{A}$'s idle periods. Leveraging equilibrium analysis of queues, we characterize the revenue rate of such a platform as a function of the price it charges its transport customers and the battery split. We further provide an algorithm to compute the optimal prices and battery split for exponentially distributed trip times, and use it to study the dependency of the optimal revenue rate and the resulting battery split on various system parameters.

We aim to extend our analysis to the case where the carsharing company $\mathbf{A}$ commands a fleet with more than one vehicle. This work has ignored the possibility that $\mathbf{A}$ maintains more than one charging depot across a city resulting in a queuing network—an interesting consideration for future work. Spatio-temporal variations in demand often lead carsharing systems to have excess cars in one location, and a paucity in another. We aim to extend our analysis, accounting for rebalancing costs among depots through appropriate incentive mechanisms, e.g., in [19]. This work only considers vehicle batteries to garner revenues from price

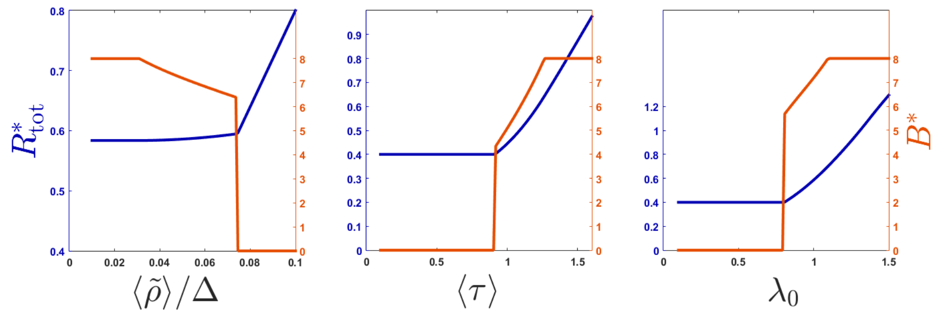

Fig. 3: Variation of the optimal revenue rate and battery split with $\langle\tilde{\rho}\rangle/\Delta$, $\langle\tau\rangle$ and λ_0. Other parameters are the same as used for Figure 2.

arbitrage; we will consider alternate grid services that the 'grid' battery might provide. Finally, we wish to utilize our framework and data from carsharing services to analyze how profitable such dual service provision will be in practice.

Appendix

Consider the optimal value functions

$$J^*_{T-1}(z, \rho_{\leq T-1}) := \underset{u}{\text{maximum}} \quad -\rho_{T-1} u, \qquad \text{subject to} \quad 0 \leq z + u \leq B', \tag{9}$$

$$\begin{aligned} &J^*_t(z, \rho_{\leq t}) \\ &:= \underset{u}{\text{maximum}} \quad -\rho_t u + \mathbb{E}\left[J^*_{t+1}(z+u, \rho_{\leq t+1}) \mid \rho_{\leq t}\right], \\ &\quad \text{subject to} \quad 0 \leq z + u \leq B'. \end{aligned} \tag{10}$$

for $t = 0, \ldots, T-2$. By [20, Proposition 1.3.1], the parametric optimizers of the above optimization problems identify the optimal policy, and the required optimal cost is given by

$$J^*(z_0) = \mathbb{E}\left[J^*_0(z_0, \rho_0)\right].$$

Since $\rho_{T-1} \geq 0$, the optimizer of (9) is given by $u^* = -z$ that yields

$$J^*_{T-1}(z, \rho_{\leq T-1}) = \rho_{T-1} z, \quad \gamma^*_{T-1}(z, \rho_{\leq T-1}) \quad = -z. \tag{11}$$

Next, we utilize (10) and backward induction to prove

$$\begin{aligned} J^*_t(z, \rho_{\leq t}) &= \rho_t z + B' \sum_{j=t}^{T-2} \mathbb{E}\left[\left(\rho_{j+1|j} - \rho_j\right)^+ \mid \rho_{\leq t}\right], \\ \gamma^*_t(z, \rho_{\leq t}) &= \begin{cases} B' - z_t, & \text{if } \rho_t \leq \rho_{t+1|t}, \\ -z_t, & \text{otherwise} \end{cases} \end{aligned} \tag{12}$$

for each $z \in [0, B']$, price sequence $\boldsymbol{\rho}$ and $t = 0, \ldots, T-2$. The rest follows from (12) with $t = 0$. Details are omitted for space constraints.

References

[1] R. Kline and T. Pinch, "Users as agents of technological change: The social construction of the automobile in the rural United States," *Technology and Culture*, vol. 37, no. 4, pp. 763–795, Oct. 1996.

[2] M. Alizadeh, H.-T. Wai, M. Chowdhury, A. Goldsmith, A. Scaglione, and T. Javidi, "Optimal pricing to manage electric vehicles in coupled power and transportation networks," *IEEE Trans. Control Netw. Syst.*, vol. 4, no. 4, pp. 863–875, Dec. 2017.

[3] A. Sundararajan, *The Sharing Economy*. Cambridge, MA: MIT Press, 2016.

[4] R. Sioshansi and P. Denholm, "The value of plug-in hybrid electric vehicles as grid resources," *Energy J.*, vol. 31, no. 3, pp. 1–23, 2010.

[5] S. Han, S. Han, and K. Sezaki, "Development of an optimal vehicle-to-grid aggregator for frequency regulation," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 65–72, Jun. 2010.

[6] E. Sortomme and M. A. El-Sharkawi, "Optimal scheduling of vehicle-to-grid energy and ancillary services," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 351–359, Mar. 2012.

[7] J. Hu, H. Morais, T. Sousa, and M. Linda, "Electric vehicle fleet management in smart grids: A review of services, optimization and control aspects," *Renew. Sustain. Energy Rev.*, vol. 56, pp. 1207–1226, Apr. 2016.

[8] G. Brandstätter, M. Kahr, and M. Leitner, "Determining optimal locations for charging stations of electric car-sharing systems under stochastic demand," *Transport. Res. B: Method.*, vol. 104, pp. 17 – 35, Oct. 2017.

[9] S. Illgen and M. Höck, "Electric vehicles in car sharing networks – challenges and simulation model analysis," *Transport. Res. D: Transport and Environ.*, vol. 63, pp. 377–387, Aug. 2018.

[10] R. F. F. Lemme, E. F. Arruda, and L. Bahiense, "Optimization model to assess electric vehicles as an alternative for fleet composition in station-based car sharing systems," *Transport. Res. D: Transport and Environ.*, vol. 67, pp. 173–196, Feb. 2019.

[11] D. R. Leonard, J. B. Tough, and P. C. Baguley, "CONTRAM: A traffic assignment model for predicting flows and queues during peak periods," Transport and Road Research Laboratory (TRRL), Wokingham, Berkshire, United Kingdom, TRRL Laboratory Report 841, Sep. 1978.

[12] R. C. Larson and A. R. Odoni, *Urban Operations Research*. Englewood Cliffs, NJ: Prentice-Hall, 1981.

[13] E. Cascetta, *Transportation Systems Engineering: Theory and Methods*. Springer, 2013.

[14] D. K. George and C. H. Xia, "Fleet-sizing and service availability for a vehicle rental system via closed queueing networks," *Eur. J. Oper. Res.*, vol. 211, no. 1, pp. 198–207, May 2011.

[15] S. de Luca and R. D. Pace, "Modelling the propensity in adhering to a carsharing system: A behavioral approach," *Transportation Research Procedia*, vol. 3, pp. 866 – 875, 2014, 17th Meeting of the EURO Working Group on Transportation, EWGT2014, 2-4 July 2014, Sevilla, Spain.

[16] S. Bose and E. Bitar, "The Marginal Value of Networked Energy Storage," *arXiv e-prints*, p. arXiv:1612.01646, Dec. 2016.

[17] S. Kara, W. Li, and N. Sadjiva, "Life cycle cost analysis of electrical vehicles in australia," *Procedia CIRP*, vol. 61, pp. 767 – 772, 2017, the 24th CIRP Conference on Life Cycle Engineering.

[18] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the lambertw function," *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, Dec 1996.

[19] S. L. Smith, M. Pavone, M. Schwager, E. Frazzoli, and D. Rus, "Rebalancing the rebalancers: optimally routing vehicles and drivers in mobility-on-demand systems," in *2013 American Control Conference*, June 2013, pp. 2362–2367.

[20] D. P. Bertsekas, *Dynamic programming and optimal control*. Athena Scientific Belmont, MA, 1995, vol. 1, no. 2.